Variable Radii Connected Sensor Cover in Sensor Networks

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One of the useful approaches to exploit redundancy in a sensor network is to keep active only a small subset of sensors that are sufficient to cover the region required to be monitored. The set of active sensors should also form a connected communication graph, so that they can autonomously respond to application queries and/or tasks. Such a set of active sensors is known as a connected sensor cover, and the problem of selecting a minimum connected sensor cover has been well studied when the transmission radius and sensing radius of each sensor is fixed. In this article, we address the problem of selecting a minimum energy-cost connected sensor cover, when each sensor node can vary its sensing and transmission radius; larger sensing or transmission radius entails higher energy cost.

For the above problem, we design various centralized and distributed algorithms, and compare their performance through extensive experiments. One of the designed centralized algorithms (called CGA) is shown to perform within an $O(\log n)$ factor of the optimal solution, where n is the size of the network. We have also designed a localized algorithm based on Voronoi diagrams which is empirically shown to perform very close to CGA, and due to its communication-efficiency results in significantly prolonging the network lifetime. We also extend the above algorithms to incorporate fault tolerance. In particular, we show how to extend the algorithms to address the minimum energy-cost connected sensor k-cover problem, in which every point in the query region need to be covered by at least k distinct active sensors. The CGA preserves the approximation bound in this case. We also propose a localized topology control scheme to preserve k-connectivity, and use it to extend the Voronoi based approach to computing a minimum energy-cost k_1-connected k_2-cover. We study the performance of our proposed algorithms through extensive simulations.

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1. INTRODUCTION

Wireless sensor networks are often deployed for passive gathering of sensor data in a geographical region. The "grand challenge" of sensor network design for data gathering activities is to maintain the fidelity of the gathered data while minimizing energy usage in the network. Energy is spent due to message transmissions by the radio interface, or due to the sensing activities by the signal processing electronics. Energy can be saved if these activities are used only to the extent absolutely needed, and no more.

Two important properties of a sensor network play critical roles in the design approach. They are *coverage* and *connectivity*. Loosely speaking, coverage describes how well sensors in the network can monitor a geographical region in question. This can include multiple parameters, such as whether every point in the region can be monitored by at least one sensor within a given confidence. The confidence typically depends on the physical distance of the point from the monitoring sensor, as distance weakens the signal and thus worsens the signal-to-noise ratio introducing measurement errors. In a simplified model, this confidence can be specified in terms of a *sensing range* [Charkrabarty et al. 2002]. Connectivity, on the other hand, simply describes the connectivity properties of the underlying network topology. It is often desirable that the network is connected. If the network is partitioned, the entire sensor network data cannot be gathered to a central decision-making node. Moreover, in some applications, the desired accuracy of sensed data and fault tolerance make it often necessary to require each point in the query region to be within the sensing region of at least k sensors. This fault tolerance is integrated into connectivity by requiring the network to be k-connected, i.e., the network remains connected even if k nodes failt.

It is expected that in most deployment scenarios, it will be cost-effective to deploy the sensors randomly in a redundant fashion ([Ye et al. 2003; Wang et al. 2003]). The sensor hardware is cheap, relative to the logistics or opportunity cost of deployment. Thus, it is useful to deploy the sensors redundantly, and employ sophisticated protocol support so that only a "minimally sufficient subset" of the sensors is actually active at a time – thus conserving energy and prolonging the sensor network lifetime. Also, in many scenarios the logistics for designed placement of sensor nodes at specific geographical locations will be very complex. Thus, in these scenarios, random deployment is the only feasible method. This means that the "minimally sufficient subset" cannot be pre-determined. The sensor nodes must be able to compute this on-line, by executing appropriate algorithms.

In this paper, our goal is to investigate such algorithms for energy-efficient connectivity and coverage. We investigate the situation where both sensing and transmission range can be varied in the sensors. This uncovers an interesting design problem, where a minimally sufficient subset of sensors must be selected along with the assignment of sensing and transmission ranges for individual sensors, such that both coverage and connectivity are guaranteed with a *minimum total* energy cost. The assumption here is that the energy cost for an individual sensor increases with higher sensing range or transmission range. This is because with a larger sensing range, more energy is needed for appropriate filtering and signal processing methods to improve the signal-to-noise ratio in order to achieve the desired confidence level. Similarly, with a larger transmission range, transmission power is to be increased to reach larger distances. It is expected that with sophisticated sensors that can control their sensing and transmission ranges, the overall energy budget of the network can be reduced relative to the case where sensors have fixed sensing and transmission ranges. Note that a similar problem has been investigated in literature by varying transmission ranges of the nodes for minimum energy topology construction in wireless ad hoc networks ([Wieselther et al. 2000; Cagalj et al. 2002; Cartigny et al. 2003; Wan et al. 2001]); however, this line of work does not involve any notion of sensing range. The model of sensors with variable sensing range has been used in [Dhawan et al. 2006; Younis et al. 2007; Cardei et al. 2005; Wu and Yang 2004], and sensors with variable sensing range are also commercially available [Osi].

The rest of the paper is organized as follows. We start in Section 2 by describing our sensor network model, and formally defining the variable radii connectivity and coverage problem addressed in this article. In the next section, we present a discussion on related work. In Section 4, we present a fully localized algorithm based on Voronoi diagrams for computing a variable radii connected sensor cover. We extend our Voronoi based approach to incoporate fault-tolerance, i.e., to compute a set of sensors that is k_1-connected and a k_2-cover. Section 6 presents centralized and distributed greedy algorithms. We present simulation results in Section 7, and end with concluding remarks in Section 8.

2. PROBLEM FORMULATION

In this section, we motivate and formulate the variable radii connected sensor cover problem addressed in this paper. We start with describing the sensor network model used in this paper.

A sensor network consists of a large number of sensors distributed randomly in a geographical region. Each sensor I has a unique ID, and is associated with a maximum sensing radius S^* and a maximum transmission radius T^*. We assume that the maximum radii associated are same for all the sensors in the network.[1] Each sensor I also chooses (or, is assigned) a sensing radius $S(I)$ $(\leq S^*)$ and a transmission radius $T(I)$ $(\leq T^*)$, such that it is capable of sensing up to a distance of $S(I)$ and can communicate directly to sensors that are within a distance of $T(I)$ units. The *assigned sensing region* $\theta(I)$ associated with a sensor I is a disk of radius $S(I)$ centered at the location of sensor I. Throughout this article, we use $d(x, y)$ to denote the euclidean distance between points x and y.

The variable radii connected sensor cover (VRCSC) problem in the above described sensor model can be informally stated as follows. Given a sensor network and a query region, select a subset of sensors with specified sensing and transmission radii, such that (a) each point in the query region can be sensed by at least one of the selected sensors, and (b) the selected sensors form a connected communication graph using their assigned transmission radii (considering only bidirectional link). Our goal is to minimize the total energy cost of the selected sensors, i.e., the sum of the sensing and communication energy costs of all the selected sensor nodes. Essentially, for a given query region in a sensor network, we wish to select

[1]This assumption is needed only for the Voronoi based approach presented in Section 4.

a subset of sensors to be powered ON and assign them sensing and transmission radii, such that the given query region is covered and the selected set of sensors form a connected communication graph. The query region can also be thought of as a surveillance region that needs to be monitored by the sensor network.

Motivation for Variable Radii. Energy is a critical resource in sensor networks. One of the key characteristics in wireless communication is that the energy consumption increases with the transmission distance. Thus, a wireless device can change its transmission range to save energy [Cagalj et al. 2002; Wan et al. 2001; Wieselther et al. 2000]. In conventional sensor design, the energy spent in sensing has an inverse relationship with the amount of signal energy received by the sensor. This is because, if the signal energy is weak, the signal to noise ratio needs to be suitably improved for reliable detection via appropriate signal processing methods.[2] Note also that the signal energy decays with distance of the sensor from the signal source according to an inverse power law. Thus, it is fair to model the energy spent in sensing as an increasing function of a power of the sensing radius. The same model is also used in [Pattem et al. 2003; Dhawan et al. 2006; Younis et al. 2007; Cardei et al. 2005; Wu and Yang 2004], and sensors with variable sensing radii are also commercially available [Osi].

Formal Problem Definition. We now formally define the variable radii connected sensor cover (VRCSC) problem. We start with a few definitions.

Definition 1. (Energy Cost) Consider a sensor I with an assigned sensing radius of $S(I)$ and a transmission radius of $T(I)$. We model the energy cost of I as $E(I) = f(S(I)) + g(T(I)) + C$, where $f(x)$ and $g(x)$ are monotonically non-decreasing functions in x, and C is a constant that represents the idle-state energy cost. □

Definition 2. ((Full) Communication Graph) Given a set of sensors M in a sensor network, the *communication graph* of M is an *undirected* graph with M as the set of vertices and an undirected edge between any two sensors if they can directly communicate with *each other* using their *assigned* transmission radii. The *full-communication graph* of a set $\mathcal{I}$ of sensors is the communication graph of $\mathcal{I}$ when each node in $\mathcal{I}$ is assigned the maximum transmission radius T^*. □

Definition 3. (Communication Distance) A path of nodes/sensors between I_i and I_j in the communication graph is called a *communication path* between the sensors I_i and I_j. The *communication distance* between two sensors I_i and I_j is the weight of the minimum node-weighted path between I_i and I_j in the communication graph, where the weight at an intermediate sensor node I is the transmission energy cost $g(T(I))$ of the sensor node. □

Definition 4. (Variable Radii Connected Sensor Cover) Consider a sensor network. Let S^* and T^* be the maximum sensing and transmission radius respectively. Given a query region R_Q in the network, a set of sensors $M = \{I_1, I_2, \ldots, I_m\}$ in the sensor network, where each sensor I_j is *assigned* a sensing radius $S(I_j)$ $(\leq S^*)$

[2] The actual relationship between the energy spent in sensing and signal energy incident on the sensor cannot be easily generalized, as it is dependent on the sensor technology and electronic circuitry for detection, but it is not important for our purposes.

and a transmission radius $T(I_j)$ $(\leq T^*)$, is said to be a *variable radii connected sensor cover* for the query region R_Q if the following two conditions hold:

(1) $R_Q \subseteq \theta(I_1) \cup \ \theta(I_2) \cup \ \ldots \theta(I_m)$, where $\theta(I_j)$ is the sensing region of I_j, i.e., a circular region of radius $S(I_j)$ centered around the sensor I_j, and

(2) the communication graph of M is connected.

A set of sensors that satisfies only the first condition is called a *variable radii sensor cover.* □

The variable radii connected sensor cover problem of computing a minimum energy-cost variable radii connected sensor cover is NP-hard as the less general problem of connected sensor cover with fixed radii is known to be NP-hard [Gupta et al. 2003].

Fault tolerance is a major conscern in sensor networks, since sensor nodes are often error prone. In order to combine fault tolerance into consideration, we give a general definition of the variable radii connected sensor cover, namely *variable radii* k_1*-connected* k_2*-cover.* Beside fault tolerance, having multiple sensors covering each point improves the accuracy of tracking, masks the false activation of sensors, and is necessary for the purposes of classification [Kumar et al. 2004].

Definition 5. (k-Connectivity) The communication graph of a given set of sensors M is k*-connected* if for any two vertices I_i and I_j in M, there are k vertex-disjoint paths from I_i to I_j. A equivalent definition is, after the removal of any $k-1$ nodes the communication graph of M is still connected. □

Definition 6. (Variable Radii k_1-Connected k_2-Cover) Consider a sensor network consisted of a set $\mathcal{I}$ of sensors and a query region R_Q. A set of sensors $M \subseteq \mathcal{I}$, $M = I_1, I_2, \ldots, I_m$, is chosen to be active, where each sensor I_j is assigned a sensing radius $S(I_j)(\leq S^*)$ and a transmission radius $T(I_j)(\leq T^*)$. M is said to be a k_1*-connected* k_2*-cover* for the query region R_Q if the following two conditions are satisfied:

(1) each point p in R_Q is covered by at least k_2 distinct sensors in M.

(2) the communication graph induced by M is k_1-connected.

□

3. RELATED WORK

Connectivity is a fundamental issue in wireless ad hoc environment, and many schemes have been addressed to conserve energy while maintaining connectivity in the network topology. One of the most related problem in the above context is the minimum connected dominating set problem [Guha and Khuller 1998]. The work in wireless network research community ([Wan et al. 2002; Das et al. 1997; Laouiti et al. 2002; Wu and Li 2001; Alzoubi et al. 2002; Chen and Liestman 2002; Wu and Dai 2003]) has primarily focused on developing energy-efficient distributed algorithms to construct a near-optimal connected dominating set. All the above works assume fixed transmission range for each sensor node. The works in [Wieselther et al. 2000; Cagalj et al. 2002; Cartigny et al. 2003; Wan et al. 2001] address the related NP-complete problem of constructing a minimum energy broadcast tree in a network, where every node can adjust its transmission power/range. Along this same line, some recent works also address the problem of fault tolerant topology control [Hajiaghayi et al. 2003; Bahramgiri et al. 2002; Li and Hou 2004; Li et al. 2003]. Of particular interest to us is the protocol in [Li et al. 2005] that proposes a cone based topology control (CBTC) scheme. The CBTC scheme is to assign the minimum transmission range to a node I such that the maximum angle between any pair of its two consecutive neighbors is at most $2\pi/3$. It is shown that the CBTC scheme preserves the connectivity of the given network. Furthermore, it is shown in [Bahramgiri et al. 2002] that CBTC actually preserves k-connectivity of the whole network, when the maximum angle between any pair of consecutive neighbors of each node is at most $2\pi/3k$. However, none of the above described works involve any notion of sensing range or coverage.

Recently, there has been a lot of research done to address the coverage problem in sensor networks. In particular, the authors in [Slijepcevic and Potkonjak 2001] design a centralized heuristic to select mutually exclusive sensor covers that independently cover the network region. In [Charkrabarty et al. 2002], the authors investigate linear programming techniques to optimally place a set of sensors on a sensor field (three dimensional grid) for a complete coverage of the field. Meguerdichian et al. ([Meguerdichian et al. 2001; Meguerdichian et al. 2001]) consider a slightly different definition of coverage and address the problem of finding maximal paths of lowest and highest observabilities in a sensor network. A localized protocol is proposed in [Yan et al. 2003] that aims at choosing minimal sensors to be active at any time point, while guaranteeing the coverage of the grid points. Some work ([Hsin and Liu 2004; Shakkottai et al. 2003; Kumar et al. 2004]) try to address the asymptotic coverage problem, in which they derive the necessary conditions such that the query region can be covered with high probability, while using simple scheduling scheme to coordinate sensor nodes duty cycles. Among them, [Kumar et al. 2004] analyzes the asymptotic coverage for the common case of k-coverage. However, all of the above works only consider fixed sensing radii. Moreover, they do not incorporate the requirement of connectivity.

Recently, researchers have also considered connectivity and coverage in an integrated platform. In particular, the authors in [Shakkottai et al. 2003] consider an unreliable sensor network, and derive necessary and sufficient conditions for the coverage of the region and connectivity of the network with high probability. The PEAS protocol [Ye et al. 2003] considers a probing technique that maintains only a necessary set of sensors in working mode to ensure coverage and connectivity with high probability under certain assumptions. Wang et al. [Wang et al. 2003] present a localized heuristic in which they use the SPAN [Chen et al. 2001] protocol to maintain connectivity, and a separate CCP protocol to maintain coverage, which can be extended for k-coverage. In our prior work [Gupta et al. 2003], we designed a greedy approximation algorithm that delivers a connected sensor cover for a sensor network with fixed transmission and sensing ranges. The above work was extended for k-coverage in [Zhou et al. 2004]. In this article, we consider the network model wherein each sensor node has the ability to adjust its transmission and sensing power/radii. We also extend our work to incorporate fault tolerance by addressing the problem of selecting k_1-connected k_2-cover sets.

4. VORONOI BASED ALGORITHM

In this section, we design a localized distributed algorithm for the variable radii connected sensor cover problem based on the computational geometric concepts of Voronoi diagram and Relative-Neighbor Graph (RNG). The developed algorithm is a localized algorithm in the sense that each sensor makes decisions based only upon local neighborhood information. Below, we recall definitions of Voronoi diagrams and Relative-Neighbor Graphs.

Definition 7. (Voronoi Diagram/Cell/Neighbor) Given n nodes in a plane, the *voronoi diagram* is defined as the partitioning of the plane into n convex polygons such that each polygon contains exactly one of the n nodes and every point in a given polygon is closer to its central node than to any other node [mat]. The *voronoi cell* of a node is the convex polygon in the voronoi diagram that contains the node. Two nodes whose voronoi cells share a common edge are called *voronoi neighbors*. □

Definition 8. (Relative Neighbor Graph (RNG)) Given nodes with uniform transmission radius T in a 2D plane, the *relative neighbor graph* is the graph where an edge exists between any two nodes u, v, *iff* (i) $d(u,v) \leq T$, and (ii) there is no node w such that $d(u,w) < d(u,v)$ and $d(v,w) < d(u,v)$. It is well-known that the relative neighbor graph is connected if the network's full-communication graph (using T as the maximum transmission radius) is also connected [Cartigny et al. 2003]. □

Definition 9. (l-hop Active Neighborhood) The l-hop active neighborhood of an active node I, denoted as $N(I,l)$, is defined as the set of active nodes that are at most at a distance of l hops from I in the unweighted full-communication graph of the entire sensor network. □

In our proposed localized algorithm, each sensor node I builds its voronoi cell based upon locations of nodes in $N(I,l)$. A low l can result in construction of inaccurate voronoi cells, since each sensor node has only limited (l-hop) information. However, a low value of l does not affect the correctness of our proposed algorithm. The constant l is chosen carefully – larger l results in better performance, but higher communication cost. For ease of presentation, we will assume that l is a constant in the rest of the discussion.

Definition 10. (Local Voronoi Cell/Neighbor) A *local voronoi cell* $LV(I)$ of a node I is a set of points p such that p is in the given query region and $d(p,I) \leq d(p,J)$ for all $J \in N(I,l)$. Note that local voronoi cells of a set of nodes in a 2D plane may not be disjoint because l may not be large enough. For a node I, the *size* of its local voronoi cell $LV(I)$ is the maximum distance of a point in $LV(I)$ from I.

A node J is a *local voronoi neighbor* of I if J is a voronoi neighbor of I in the voronoi diagram over the set of nodes $N(I,l)$. Note that the local voronoi neighbor relationship is not symmetric, i.e., I may not be a local voronoi neighbor of J even if J is a local voronoi neighbor of I. We use $LN(I)$ to denote the set of local voronoi neighbors of I. □

The following method of assignment of radii to a set of active sensor nodes in a sensor network forms the core of our Voronoi Based algorithm.

V-R Assignment of Radii. Consider a set of active sensors A in a sensor network. Let the set of sensor nodes whose maximum sensing region intersects with the given query region be M. The *V-R assignment* of sensing and transmission radii is defined as follows. Each sensor node I in M is assigned a sensing radius equal to the size of its local voronoi cell or the maximum sensing radius, whichever is smaller. Each sensor node I in M is assigned a transmission radius equal to the maximum distance over all its neighboring nodes in the RNG graph of M. All active nodes that are not in M are assigned zero sensing and transmission radius. The following theorem shows that the V-R assignment ensures coverage and connectivity of the query region.

THEOREM 1. *Given a set of active sensors A and a query region in a sensor network, such that the query region is covered by the union of the maximum sensing regions of nodes in A, the V-R assignment of radii ensures coverage of the query region.*

Let the set of active sensor nodes whose maximum sensing region intersects with the query region be M. If the full-communication graph of M is connected, then the V-R assignment of transmission radii ensures connectivity of M.

PROOF. It is easy to see that $(V(I) \cap R_Q) \subseteq LV(I)$, where $V(I)$ is the voronoi cell of I and R_Q is the query region. Consider a point p in the query region, and let I_p be the active sensor node that is closest to p. Now, $p \in V(I_p)$ and hence, $p \in LV(I_p)$. Since p is covered by the maximum sensing region of at least one active sensor node, it is covered by the maximum sensing region of I_p, and hence, the assigned sensing region of I_p covers p.

As RNG is guaranteed to be connected, the V-R assignment ensures connectivity of M. ∎

Voronoi Based Algorithm Description. The V-R assignment of sensing and transmission radii is key in the design of our Voronoi Based algorithm. Informally, the Voronoi Based algorithm works as follows. We start with all sensors in the network as active nodes, and use the V-R assignment method to assign their sensing and transmission radius. At each stage, certain sensor nodes become inactive, and the assignment of sensing and transmission radii is redone for the remaining active nodes. A sensor node is chosen to become inactive only if the remaining active sensors are capable of covering the query region and maintain connectivity of their communication graph. We use an appropriately defined concept of "benefit" to choose the best sensor nodes to become inactive. The algorithm terminates when no more sensors can be made inactive. In the end, the set of active sensor nodes form the desired VRCSC solution. Formally, our proposed Voronoi Based algorithm consists of the following steps.

(1) Initially, each sensor node in the sensor network is active, and gathers locations of all the nodes in the l-hop active neighborhood.

(2) Each active sensor node computes its local voronoi cell, and the neighbors in the RNG over active nodes. It uses the V-R assignment method to assign itself a sensing and a transmission radius.

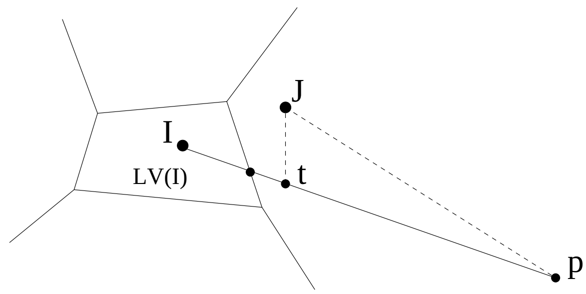


Fig. 1. Proof of Theorem 2

(3) Each node I computes its *sleeping benefit* (formally defined later), which is the decrease in the total energy cost of the "local" active sensors if I is inactivated.

(4) A sensor node I is considered *removable*, if it satisfies the following two conditions.
 —For every pair of communication neighbors of I, there exists a communication path P in the full-communication graph of $N(I, l)$, such that all the intermediate nodes in P have a higher node-ID than that of I. This condition ensures connectivity of active nodes, if I is made inactive [Wu and Dai 2003].
 —The region $(LV(I) \cap \theta(I))$ is covered by the union of the maximum sensing regions of the local voronoi neighbors of I. We show in Theorem 2 that the above condition ensures coverage of the query region, if I is made inactive.

(5) If I is removable and has the most *sleeping benefit* among all its local voronoi neighbors, then I becomes inactive.

(6) Go to Step 2.

The above described algorithm can be easily implemented in a distributed setting, where the communication model is reliable. To ensure correctness in an unreliable communication model, we need to add certain tedious steps as discussed later. This completes the description of the algorithm.

Coverage Guarantee. Now, we show that the above described algorithm maintains coverage of the query region, if the query region was initially covered by the active sensors. We use $\theta^*(I)$ to represent the maximum sensing region (corresponding to the maximum sensing radius S^*) of I. Also, recall that $LN(I)$ is the set of local voronoi neighbors of I. We start with a lemma.

LEMMA 1. *Let I be an active sensor, and $\theta(I)$ be the sensing radius assigned by the V-R assignment method (step (2) of the Voronoi Based algorithm). If $LV(I) \cap \theta(I) \subseteq \bigcup_{j \in LN(I)} \theta^*(j)$, then $\theta^*(I) \subseteq \bigcup_{j \in LN(I)} \theta^*(j)$.*

PROOF. Consider an arbitrary point p in $\theta^*(I)$. We show that $p \in \bigcup_{j \in LN(I)} \theta^*(j)$. Let us consider two cases depending on whether $LV(I)$ contains p.

First, consider the case when $p \in LV(I)$. In V-R assignment of radii, either $LV(I) \subseteq \theta(I)$ or $\theta(I) = \theta^*(I)$. Thus, we have $p \in \theta(I)$. Hence, $p \in \bigcup_{j \in LN(I)} \theta^*(j)$.

Now, consider the case when $p \notin LV(I)$. As shown in Figure 1, there exists a point $t \notin LV(I)$ on the line segment $\overline{pI}$. Also, there is a sensor $J \in LN(I)$, such that $d(J,t) < d(I,t)$. Now,

$$\begin{aligned} d(J,p) &< d(t,p) + d(t,J) \\ &< d(t,p) + d(t,I) \\ &= d(p,I) \\ &< S^* \end{aligned}$$

Thus, $p \in \theta^*(J)$, and $p \in \bigcup_{j \in LN(I)} \theta^*(j)$. ∎

THEOREM 2. *Given a set of active sensors A and a query region in a sensor network, such that the query region is covered by the union of the maximum sensing regions of nodes in A, the Voronoi Based algorithm ensures coverage of the query region.*

PROOF. We know by Theorem 1 that the initial V-R assignment ensures coverage of the query region. Below, we show that at any stage of the algorithm and for every point p in the query region, there is an active sensor node H covering p using its maximum sensing radius that cannot be inactivated.

Let $C(p)$ denote the set of active sensors that can cover a point p using their maximum sensing regions. Consider a point p in the query region such that $C(p) \neq \emptyset$. Let H be the sensor node with minimum sleeping benefit in $C(p)$. We show that the sensor node H will not be inactivated by the Voronoi Based algorithm. Let us assume the contrary that the sensor node H is inactivated, which means that $(LV(H) \cap \theta(H)) \subseteq \bigcup_{j \in LN(H)} \theta^*(j)$ and H's sleeping benefit is more than that of any sensor in $LN(H)$. From Lemma 1, we know that there exists a sensor $J \in LN(H)$ such that $p \in \theta^*(J)$. Thus, $J \in C(p)$ and J's sleeping benefit is less than that of H, which yields a contradiction. ∎

Calculating Sleeping Benefit. The *sleeping benefit* $B(I)$ of an active node I is defined as the decrease in total energy cost of the set of active sensors in the networks due to inactivation of the node I. Note that when a node I is inactivated, only the nodes J that consider I as a local voronoi neighbor need to increase their assigned sensing radius. Moreover, only the nodes H that are in the 1-hop communication neighborhood of I need to possibly increase their transmission radius due to inactivation of I. Thus, the sleeping benefit $B(I)$ of a node I can be computed as follows.

$$B(I) = E(I) - \sum_{J:I \in LN(J)} (f(S_{new}(J)) - f(S(J))) - \sum_{H \in N(I,1)} (g(T_{new}(H)) - g(T(H))),$$

where $S(X)$ and $T(X)$ are the current sensing/transmission radii of a node X, and $S_{new}(X)$ and $T_{new}(X)$ are the new sensing/transmission radii of a node X after inactivation of node I. A node I can compute the second term of the above expression using either the $2l$-hop neighborhood information or the set of local voronoi diagrams of all nodes J that considers I as its local voronoi neighbor.[3] Similarly, the third term in the above expression can be computed using the 2-hop neighborhood information of I.

Node Failures. Lastly, we need to consider the situation when a sensor dies due to complete depletion of battery power. To guarantee the connectivity and coverage, a `dying` sensor, I, broadcasts a `wake-up` message to arouse all the nodes in its l-hop neighborhood, which in turn run the Voronoi Based algorithm to assign themselves transmission and sensing radii appropriately (or to remain in inactive mode). When the network is dense, and l-hop neighborhood is relatively large, we propose to use the distributed priority algorithm of [Zhou et al. 2004] to accelerate the speed of the local recovery. In particular, the nodes receiving the `wake-up` message run the distributed priority algorithm prior to the Voronoi Based algorithm to speed up the recovery and save reconstruction cost. If any aroused node finds its local voronoi cell can not be covered, it in turn sends a `wake-up` message to its l-hop neighbors, till the query region is covered.

5. VORONOI BASED ALGORITHM FOR K_1-CONNECTIVITY K_2-COVERAGE

In this section, we extend the Voronoi Based Algorithm to solve the minimum energy-cost k_1-connected k_2-cover problem. We start with describing localized k-connectivity preserving topology control schemes which are used to extend the Voronoi Based algorithm for variable radii k_1-connected k_2-cover problem. In Section 5.2, we present the generalized Voronoi Based algorithm.

5.1 k-Connectivity Preserving Topology Control

One of the major components of the generalized Voronoi Based algorithm is to preserve k-connectivity. In this section, we present topology control strategies to delete nodes and edges in the network, while preserving k-connectivity of the remaining network. We would use the results presented in this section to design the generalized version of Voronoi Based algorithm in Section 5.2.

Topology Control by Deletion of Edges. In this section, we generalize the RNG structure to the k-RNG structure [Jaromczyk and Toussaint 1992], which allows us to delete longer edges in the graph in a distributed and localized manner while preserving k-connectivity of the graph. Deletion of longer edges allows us to reduce the transmission powers of the nodes in the network, and thus, reducing the total energy requirement of the network while preserving the desired k-connectivity requirement.

DEFINITION 11. (k^{th} Relative Neighbor Graph (k-RNG)) Given a network of n nodes with uniform transmission radius T, the k^{th} *relative neighbor graph* is the network communication graph where an edge exists between two nodes u and v *iff* (i) $d(u, v) \leq T$, and (ii) there are at most $(k-1)$ nodes w that satisfy the condition $d(u, w) < d(u, v)$ and $d(v, w) < d(u, v)$ simultaneously. An example is shown in figure 2. □

[3] In our simulations, a node I approximates the sleeping benefit $B(I)$ by using only its own local voronoi diagram.

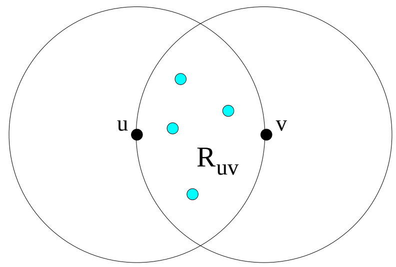
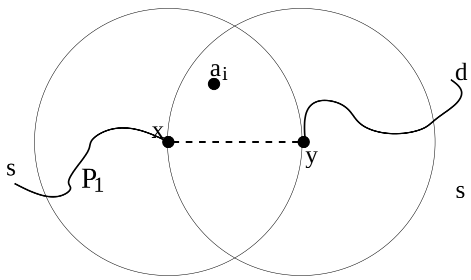
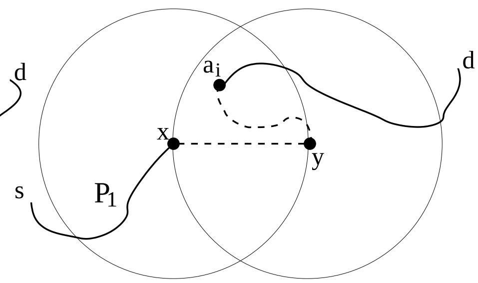


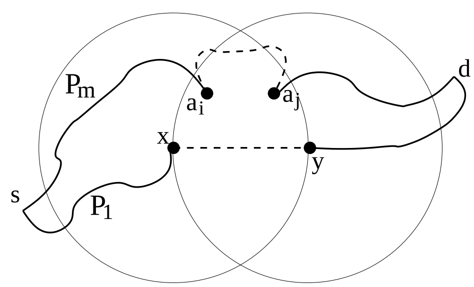
Fig. 2. k-RNG example. (u, v) is a k-RNG edge only if there exist less than k nodes within the area R_{uv}.



(a) Case 1: There exists a node a_i that is not contained in any path.



(b) Case 2: There exists a node a_i on path P_1.



(c) Case 3: There exist at least two nodes a_i and a_j on a single path P_m.

Fig. 3. Three cases in the proof of Theorem 3.

THEOREM 3. *Given a network of nodes with uniform transmission radius T, if the network's full-communication graph (using T as the maximum transmission radius) is k-connected, then the k-RNG is also k-connected.*

PROOF. Lets consider two nodes x and y such that there are at least k nodes $a_1, a_2, \ldots, a_k$ that satisfies the condition $d(x, a_i) < d(x, y)$ and $d(y, a_i) < d(x, y)$ simultaneously. Let the full-communication graph of the network be G, and let G'

be the graph G without the edge (x, y). Below, we show that G' is k-connected, assuming G is k-connected.

Consider an arbitrary pair of nodes s and d in G. Let $P_1, P_2, \ldots, P_k$ be the k node-disjoint paths between s and d in the graph G. We try to show that there exist k node-disjoint paths between s and d in G' also. If (x, y) does not belong to any P_i $(1 \leq i \leq k)$, then s and d trivially have k node-disjoint paths in G'. Without loss of generality, let us assume that (x, y) belongs to P_1. Now, there are three cases:

—There is a node a_i $(1 \leq i \leq k)$ that is not contained in any of the other paths $P_2, P_3, \ldots, P_k$. See Figure 3 (a). In this case, the edge (x, y) in P_1 can be replaced by (x, a_i, y) to yield $P_1^{'}$, and the set of k node-disjoint paths in G' connecting s and d are $P_1^{'}, P_2, P_3, \ldots, P_k$.

—There is a node $a_i (1 \leq i \leq k)$ that is contained in P_1. See Figure 3 (b). In this case, the path P_1 can be changed to yield a shorter path $P_1^{'}$ which is node-disjoint from all other paths $P_2, P_3, \ldots, P_k$. If P_1 is of the form $(s, \ldots, x, y, \ldots, a_i, \ldots, d)$, then $(s, \ldots, x, a_i, \ldots, d)$ can be chosen as $P_1^{'}$. Similarly, if P_1 is of the form $(s, \ldots, a_i, \ldots, x, y, \ldots, d)$, then $(s, \ldots, a_i, y, \ldots, d)$ can be chosen as $P_1^{'}$.

—There are two nodes a_i and a_j that are contained in the same path P_m $(2 \leq m \leq k)$. See Figure 3 (c). In this case, P_1 and P_m can be changed to yield two node-disjoint paths that are also node-disjoint from other paths. In particular, if P_1 is of the form $(s, P_1^{sx}, x, y, P_1^{yd}, d)$ and P_m is of the form $(s, P_m^{sa_i}, a_i, P_m^{a_i a_j}, a_j, P_m^{a_j d}, d)$, then we can construct two paths $P_1^{'} = (s, P_1^{sx}, x, a_j, P_m^{a_j d}, d)$, and $P_m^{'} = (s, P_m^{sa_i}, a_i, y, P_1^{yd}, d)$. It is easy to see that the set of k paths $P_1^{'}, P_2, \ldots, P_l^{'}, \ldots, P_k$ exist in G' and are node-disjoint.

Note that the above three cases cover all possibilities. Thus, the above analysis shows that G' is k-connected.

Note that in the above analysis, the new edges introduced in the paths connecting s and d are strictly shorter than (x, y). Thus, to show that the k-RNG graph is k-connected, we can apply the above analysis for one edge removed from G at a time, in the descending order of the edge lengths. □

One of the other distributed and localized schemes proposed in the literature for transmission power control while preserving k-connectivity is the CBTC [Bahramgiri et al. 2002] (cone based topology control) approach. In CBTC approach, each node u picks the minimum transmission radius t_u such that there is a node w with $d(u, w) < t_u$ in *every* cone of angle $2\pi/3k$ around u. If no such radius t_u exists for a node u, then u picks the maximum transmission radius. It is shown in [Bahramgiri et al. 2002] that the resulting graph considering only the undirected edges is k-connected. Below, we show that the k-RNG structure is actually a subgraph of the graph generated by CBTC approach in unit-disk graphs. Thus, k-RNG is more energy efficient than CBTC.

THEOREM 4. *Consider a network of nodes with uniform transmission power T. The k-RNG is a subgraph of the graph resulting from CBTC approach.*

PROOF. We prove the theorem by showing that if an edge (u, v) does not exist in CBTC graph, then (u, v) is not in k-RNG. If $d(u, v) > T$, then the claim is

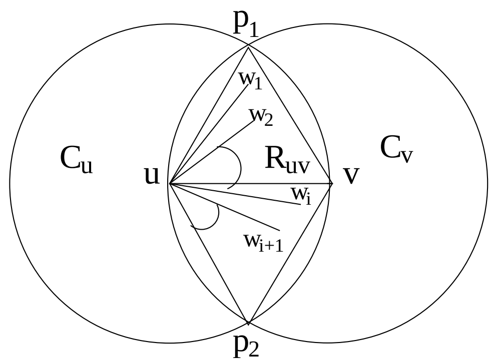


Fig. 4. Proof of Theorem 4. If (u, v) is not a CBTC edge, then there are at least k nodes in the region R_{uv}.

trivially true. Thus, $d(u, v) \leq T$, and the edge (u, v) does not exist in CTBC graph due to reduction in the transmission radius of u and/or v. Let t_u and t_v be the transmission randii of u and v respectively resulting from CBTC approach. Since (u, v) is not an edge in CBTC graph, we know that either $t_u < d(u, v)$ or $t_v < d(u, v)$. Without loss of generality, let us assume that $t_u < d(u, v)$.

Now, consider the circles C_u and C_v with centers u and v respectively and radii $d(u, v)$, and the intersection region R_{uv} of the circles C_u and C_v as shown in Figure 4. Let p_1 and p_2 be the points of intersection of the two circles. Note that $\angle p_1 u p_2 = 2\pi/3$ and $d(u, p_1) > t_u$. By definition of CBTC, since there is a node w in every cone of angle $2\pi/3k$ around u such that $d(u, w) < t_u$, there are at least k nodes $w_1, w_2, \ldots, w_k$ in the cone confined by segments $u\bar{p}_1$ and $u\bar{p}_2$ such that $d(u, w_i) < t_u$ for each w_i. The above implies that there are k nodes in the region R_{uv}, and hence, (u, v) is not an edge in k-RNG. ∎

Topology Control by Deletion of Nodes. In [Wu and Dai 2003], Wu and Dai propose several schemes for distributed computation of connected dominating sets. The general strategy of their schemes was to delete nodes I that satisy the condition that for every pair of neighbors u and v there is a path (called a *replacement* path) containing nodes with priority (which could be the unique node ID) higher than that of I. The above condition can be tested in a distributed and localized manner by requiring the replacements paths to exist in the d-hop neighborhood of each node I. They show that after deletion, the remaining nodes form a connected dominating set. Below, we generalize their approach to construct a k-connected dominating set in a distributed manner. In particular, we propose inactivation of a nodes I that satisfy the below defined k-delNode condition, and show that the set of remaining active nodes form a k-connected dominating set.[4]

DEFINITION 12. (k-delNode condition:) A node I is said to satisfy the *k-delNode condition* if for every pair of active neighbors u and v of I, there exists k node-disjoint paths $P_1, P_2, \ldots, P_k$ containing only higher-priority (relative to I's priority)

[4]For application of the k-delNode condition to the problem addressed in this article, we do not need the dominating property of the non-deleted nodes.

active intermediate nodes. □

THEOREM 5. *Given that the full-communication graph of a given set of active sensors is k-connected. After iterative inactivation of nodes that satisfy the k-delNode condition in the full-communication graph, the full-communication graph of the remaining active nodes is still k-connected.* □

In work done concurrently with ours, Wu and Dai [Dai and Wu 2005] have shown an even stronger result that the remaining active nodes that do not satisfy the k-delNode[5] form a k-connected k-dominating set. We refer the reader to [Dai and Wu 2005] for a proof of the above theorem.

5.2 Generalized Voronoi Based Algorithm

In this subsection, we extend the Voronoi Based algorithm to variable radii k_1-connected k_2-cover problem. Basically, we use the concept of k-RNG and the condition of k-delNode described in Section 5.1 to address the connectivity issue; and the concept of k^{th}-order voronoi diagram described below to address the coverage issue. First, we review some definitions related to k^{th}-order voronoi diagrams.

DEFINITION 13. (k^{th}-order Voronoi Diagram/Cell/Neighbor) Given n nodes in a plane, the k^{th}-*order voronoi diagram* is defined as the partitioning of the plane into regions that have the same set of k nearest nodes [O'Rourke 1998]. The k^{th}-*order voronoi cell* of a node I is defined as the union of the regions that have I as one of their k nearest nodes. In other words, for any point p inside the k^{th}-order voronoi cell of I, there are less than k other nodes that are closer to p than I. Two nodes are called k^{th}-*order voronoi neighbors* if their k^{th}-order voronoi cells intersect or share common edge. □

DEFINITION 14. (k^{th}-order Local Voronoi Cell/Neighbor) The k^{th} *order local voronoi cell* $LV(I)$ of a node I is the k^{th}-order voronoi cell of I in the k^{th}-order voronoi diagram over the set of nodes $N(I,l)$. That is, for any point $p \in LV(I)$, there exist at most $k-1$ nodes J in $N(I,l)$ such that $d(p,J) < d(p,I)$.

A node J is a k^{th}-*order local voronoi neighbor* of I if J is a k^{th}-order voronoi neighbor of I in the voronoi diagram over the set of nodes $N(I,l)$. Note that the k^{th}-order local voronoi neighbor relationship is not symmetric. We use $LN(I)$ to denote the set of k^{th}-order local voronoi neighbors of I. □

For any k, the k^{th}-order voronoi diagram over $N(I,l)$ can be calculated using the arrangement of planes tangent to the paraboloid above the nodes of $N(I,l)$ in time $O(|N(I,l)|^3)$ [O'Rourke 1998]. In our simulations, we use the polygon clipping method [Foley et al. 1990] to calculate the k^{th}-order local voronoi cell of I.

Generalized Algorithm. Using the above defined concepts relating to k^{th}-order voronoi diagrams, and the k-connectivity preserving scheme, the Voronoi Based algorithm generalizes to the k_1-connected k_2-cover problem naturally. In particular, we modify the V-R assignment by requiring each sensor node to cover its k_2^{th}-order local voronoi cell, and support all its edges in the k_1-RNG graph. That is, each sensor I is assigned a sensing radius equal to the smaller one of the size of its

[5]They refer to the k-delNode condition as the k-coverage condition

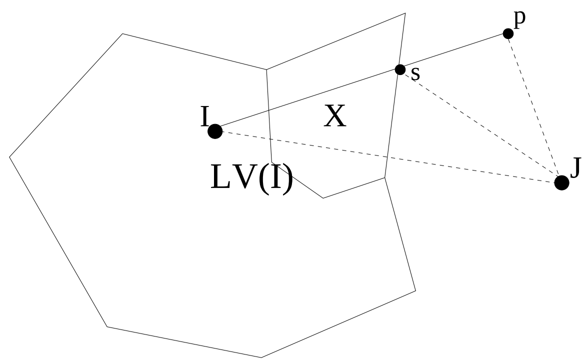


Fig. 5. Proof of Lemma 3

k_2^{th}-order local voronoi cell and the maximum sensing radius, a transmission radius equal to its longest adjacent edge in the k_1-RNG graph.

Moroever, a sensor I is considered removable only if the following two conditions are satisfied:

—Its k_2^{th}-order local voronoi cell can be k_2-covered by the union of the maximum sensing regions of the k_2^{th}-order local voronoi neighbors of I.

—I satisfies the k_1-delNode condition as described in Section 5.1.

A sensor inactivates itself when its sleeping benefit is the maximum among all its k_2^{th}-order local voronoi neighbors.

k_2-Coverage Guarantee. We show that the above generalization of the Voronoi Based algorithm ensures k_2-coverage of the given query region.

THEOREM 6. *Given a set of active sensors A and a query region in a sensor network, such that the query region is k-covered by the union of the maximum sensing regions of nodes in A, the V-R assignment of radii ensures k-coverage of the query region.*

PROOF. It is easy to see that $(V(I) \cap R_Q) \subseteq LV(I)$, where $V(I)$ is the k^{th}-order voronoi cell of I, R_Q is the query region, and $LV(I)$ is the k^{th} order local voronoi cell of I. Consider a point p in the query region, and let $\mathcal{I}_p$ be the k nearest active sensor nodes to p. Now, for any $I \in \mathcal{I}_p$, $p \in V(I)$ and hence, $p \in LV(I)$. Since p is covered by the maximum sensing region of at least k active sensor nodes, it is covered by the maximum sensing region of each node in $\mathcal{I}_p$, and hence, it is covered by the assigned sensing region of each node I in $\mathcal{I}_p$. ∎

LEMMA 2. *Consider the k^{th}-order local voronoi cell $LV(I)$ of a sensor node I. For any point $p \in LV(I)$, the line segment $\overline{pI}$ lies completely within $LV(I)$.*

PROOF. Let us assume that there exists a point $q \in \overline{pI}$, such that $q \notin LV(I)$. Then there must exist a node J, such that $d(p, J) > d(p, I)$ and $d(q, J) < d(q, I)$. Now, according to triangular inequality $d(p, J) < d(p, q) + d(q, J)$, which gives $d(p, J) < d(p, q) + d(q, I) = d(p, I)$ — a contradiction. ∎

LEMMA 3. *Let I be an active sensor, and $\theta(I)$ be the sensing region assigned by the V-R assignment. If $LV(I) \cap \theta(I)$ is k-covered by $\bigcup_{j \in LN(I)} \theta^*(j)$, then $\theta^*(I)$ is also k-covered by $\bigcup_{j \in LN(I)} \theta^*(j)$. Here, $LV(I)$ is the k^{th}-order local voronoi cell of I.*

PROOF. We show that any arbitrary point p in $\theta^*(I)$ is covered by the maximum sensing region of at least k distinct sensor nodes in $LN(I)$. We consider two cases depending on whether p is in $LV(I)$.

If $p \in LV(I)$, then $p \in \theta(I)$. Thus, $p \in (LV(I) \cap \theta(I))$ and hence, is k-covered by $\bigcup_{j \in LN(I)} \theta^*(j)$.

Let us now consider the case when $p \notin LV(I)$. From Lemma 2, we know the line segment $\overline{pI}$ intersects the border of $LV(I)$ at only one point s, as illustrated in Figure 5. Define *subcell* as the region that has the same k nearest nodes, and we denote the first subcell that $\overline{pI}$ traverses inside $LV(I)$ as X; the set of k nearest nodes relating to X as N_X (note that $I \in N_X$). Because s is at the border of $LV(I)$ and X, there exists a node $J \notin N_X$ such that $d(s, J) = d(s, I)$, and thus J is in $LN(I)$ according to the definition of $LN(I)$. Hence $d(p, J) < d(p, s) + d(s, J) = d(p, s) + d(s, I) = d(p, I) \leq S^*$. Also, for any node $u \in N_X$, we know $d(s, u) \leq d(s, J)$. Thus,

$$
\begin{aligned}
d(p, u) &< d(s, u) + d(p, s) \\
&\leq d(s, J) + d(p, s) \\
&= d(s, I) + d(p, s) \\
&= d(p, I) \\
&\leq S^*
\end{aligned}
$$

Thus, p lies in the maximum sensing region of at least k distinct nodes in $LN(I)$, that is, node J and the nodes in set $N_X - I$. ∎

THEOREM 7. *Given a set of active sensors A and a query region in a sensor network, such that the query region is k-covered by the union of the maximum sensing regions of nodes in A, the k^{th}-order Voronoi Based algorithm ensures k-coverage of the query region.*

PROOF. We showed in Theorem 6 that the V-R assignment preserves the k-coverage of the query region. Below, we show that at any stage of the algorithm, for every point p in the query region, there are at least k distinct active sensor nodes covering p using their maximum sensing region that cannot be inactivated.

Let $C(p)$ denote the set of active sensors that can cover a point p using their maximum sensing regions. Consider a point p in the query region such that $|C(p)| \geq k$. Let I be a sensor node in $C(p)$ such that its sleeping benefit is more than the sleeping benefit of at most $k-1$ other sensor nodes in $C(p)$. We show that the sensor node I will not be inactivated by the Voronoi Based algorithm. Let us assume the contrary that the sensor node I is inactivated, which means that $LV(I) \cap \theta(I)$ is k-covered by $\bigcup_{j \in LN(I)} \theta^*(j)$, and the sleeping benefit of I is maximum among all nodes in $LN(I)$. From Lemma 3, there is a set of nodes $\mathcal{H} \subseteq LN(I)$ such that $|\mathcal{H}| = k$ and each sensor node in $\mathcal{H}$ covers p with its maximum sensing region.

Thus, $\mathcal{H} \subseteq C(p)$. Also, since $\mathcal{H} \subseteq LN(I)$, I's sleeping benefit is more than the sleeping benefit of any node in H. Thus, I's sleeping benefit is more than at least k other sensors in $C(p)$, which contradicts our hypothesis. ∎

k_1-Connectivity Guarantee. Theorem 5 states that removal of nodes that satisfy the k_1-delNode condition preserves the k_1-connectivity of the full-communication graph of the remaining nodes. Also, from Theorem 3 the V-R assignment of radii preserves the k_1-connectivity, the solution returned by the Voronoi Based algorithm is k_1-connected.

5.3 Relaxation of Assumptions

The Voronoi Based Approach presented in the previous subsection appears to use a set of idealized assumptions. We argue below how such assumptions can be relaxed and the techniques can be applied to practical cases. Note that the discussion below also applies to the basic Voronoi Based Approach.

Circular Sensing Range. Our Voronoi Based approach assumes that each sensor has the same circular, maximum sensing region. However, in reality, the maximum sensing regions of different nodes may not be identical. Moreover, each sensing region may not be even circular. This may be true even when a homogenous network is used. Difference in ranges can result from noise properties, occlusion etc. In a general scenario, each sensor node has associated with it h different sensing regions (not necessarily circular) each with an associated energy cost. Our designed Voronoi Based algorithm is still applicable in this general scenarion, by choosing the minimum-energy sensing region that contains the local voronoi cell at any stage.

Circular and Uniform Transmission Ranges. The relative neighborhood graph (RNG) preserves connectivity only for the case of unit-disk graphs, i.e., when the transmission range is uniform and circular. However, in general a sensor network may not exhibit the unit-disk property because of irregularity in radio propagation, impracticality of a perfectly omnidirectional antenna, etc. Thus, we need to generalize the RNG definition for general (not unit-disk) network graphs as follows.

Definition 15. (General Relative Neighbor Graph) Given n nodes in a 2D plane, the Relative Neighbor Graph is the graph where an edge exists between any two nodes u, v, *iff* the communication link between u and v exists, and there exist no other node w that satisfies the following three conditions: (i) $d(u,w) < d(u,v)$, (ii) $d(v,w) < d(u,v)$, and (iii) edges (u,w) and (v,w) exist. □

This above definition of RNG preserves connectivity of the original graph. The definition of k-RNG can be similarly generalized. Note that Theorem 5 holds even for non unit-disk graphs. Thus, our process of deletion of nodes can still be applied for non-unit-disk graphs.

Error Free Transmissions. Below, we discuss the issues that arise in an unreliable communication model, and propose solutions to handle them.

The first problem in an unreliable communication model occurs if a node I doesn't have the updated benefit (which is sent in a message) of J, one of its local voronoi neighbors. In such a case, the second condition of removability could result in a cyclic condition in a distributed setting, and two *mutually* local voronoi neighbors

I and J may both delete themselves and thus, possibly render the query region uncovered. To prevent such a scenario from happening we require the following. A sensor I that wishes to inactivate itself, sends an inquiry to each of its local voronoi neighbors; the node I enters sleeping mode only after it has received positive confirmation from all of its local voronoi neighbors. Inquiries are resent on failures, and a sensor node that sends a positive confirmation assumes the inquirer I is inactive from then on.

The second problem arises because a sensor node I may not be able to accurately compute its $N(I, l)$, the active l-hop neighborhood, because of message losses. In particular, a node may not know which neighboring nodes are active or inactive. We solve this problem by requiring each active sensor to send a periodic `hello` message to its l-hop neighbors. By default, a node I assumes that each node J in the l-hop neighborhood is inactive, unless it receives a `hello` message from J. This results in an *underestimation* of $N(I, l)$ due to possible message losses. Underestimation of $N(I, l)$ only results in overestimation of $LV(I)$, and hence, overestimation of the assigned sensing radii. Therefore, the claims of Theorem 1 and Theorem 2, i.e., the coverage guarantee claims are not affected. The inaccuracy of neighborhood information doesn't cause any problems in maintenance of connectivity of the active nodes, as long as each node *initially* start with accurate information of one-hop communication neighbors and the active neighborhood nodes are eventually discovered.

6. GREEDY ALGORITHM

In this section, we present a greedy algorithm for the variable radii connected sensor cover problem. We present a centralized as well a distributed version of the algorithm. In contrast with the Voronoi Based algorithm, the centralized version of the greedy algorithm provably delivers a VRCSC whose total energy cost is at most $O(r \log n)$ times the optimal energy cost. Here, r is the link radius of the sensor network (defined later) and n is the total number of sensors in the entire network. The distributed version of the greedy algorithm empirically performs close to the centralized version, but incurs higher construction cost compared to the Voronoi Based algorithm due to the size of the messages. Moreover, for the greedy algorithm, we need to make an assumption that each sensor has only a finite number of choices for the sensing radii. In particular, we assume that each sensor I chooses from h sensing radii $S_1, S_2, \ldots, S_h = S^*$. The greedy algorithm presented here is a generalization of the greedy approximation algorithm presented in [Gupta et al. 2003] for the fixed radii version of the problem. We start with describing the centralized version of the greedy algorithm.

Basic Idea. Informally, the proposed greedy algorithm works as follows. The algorithm maintains a set of selected sensors M along with their assigned transmission and sensing radii, and increases the covered region while keeping connectivity of M. At each stage, we either add to M a "path" of sensors or increase the sensing radius of a sensor in M, whichever gives the maximum "benefit." The algorithm terminates when the given query region is completely covered by the assigned sensing regions of the sensors in M. A more formal and complete description of the algorithm is given below. We first start with a few more definitions.

DEFINITION 16. (Candidate Sensor; Candidate Path) Let M be the set of sensors already selected by the algorithm. A sensor c is called a *candidate sensor* if $c \notin M$ and there is a sensor m in M such that $d(c, m) < S^* + S(m)$. In other words, a sensor c is a candidate sensor if $c \notin M$ and its maximum sensing region (corresponding to the sensing radius S^*) intersects with the assigned sensing region ($\theta(m)$) of some sensor m in M.

A *candidate path* is a sequence/path of sensors $<p_0, p_1, \ldots, p_l>$ such that p_0 is a candidate sensor, $p_l \in M$, $p_i \notin M$ for $i < l$, and the sequence of sensors forms a communication path in the full-communication graph of the entire sensor network. Also, to ensure that the sequence of sensors P forms a communication path with minimum transmission energy cost, we make the following assignment of radii.

$$\begin{aligned} T(p_0) &= d(p_0, p_1) \\ T(p_i) &= \text{Max}(d(p_i, p_{i-1}), d(p_i, p_{i+1}) \quad \forall\, 0 < i < l \\ T(p_l) &= \text{Max}(d(p_l, p_{l-1}), T(p_l)) \\ S(p_i) &= 0 \ \text{ for } 0 < i < l \end{aligned}$$

In addition, the sensing radius of the candidate sensor p_0 is chosen to maximize the benefit of the candidate path (defined later). The sensing radius of p_l, which is in M, is kept unchanged. □

DEFINITION 17. (Subelement; Valid Subelement) Recall that each sensor has a choice of h possible sensing regions (corresponding to the h different sensing radii). A *subelement* is a set of points. Two points belong to same subelement if and only if they are covered by the same set of possible sensing regions. If a subelement intersects with the given query region, then it is called a *valid subelement*. □

DEFINITION 18. (Benefit of a Candidate Path) Benefit of a candidate path P with respect to M, an already selected set of sensors, is defined as the number of valid subelements newly (not covered by M) covered by P divided by the increase in energy cost of M due to addition of P. More formally, the benefit of a candidate path P with respect to a set of selected sensors M is:

$$\frac{V(M \cup P) - V(M)}{E(M \cup P) - E(M)},$$

where $V(\mathcal{I})$ is the number of valid subelements covered by a set of sensors $\mathcal{I}$, and $E(\mathcal{I})$ is the total energy cost of $\mathcal{I}$. □

DEFINITION 19. (Optimal Incremental Benefit) Let M be the set of sensors already selected by the greedy algorithm, and m be a sensor node in M with an assigned sensing radius of $S(m)$. The *incremental benefit* of increasing m's sensing radius from $S(m)$ to $S'(m)$ is defined as the number of valid subelements newly (not covered by M) covered by the increased sensing region $\theta'(m)$ divided by the increase in energy cost of m. The sensing radius $S'(m)$ of m that results in the maximum incremental benefit is called the *optimal incremental radius* of m with respect to M, and the corresponding incremental benefit is called the *optimal incremental benefit* of m. □

Centralized Greedy Algorithm. We now give a formal and complete description of the Centralized Greedy Algorithm. Initially, M consists of an arbitrary sensor I whose minimum sensing region (S_1) intersects with the given query region. The sensor I's sensing radius is set to the minimum and its transmission radius is set to zero. At each subsequent stage, the algorithm finds the candidate path $\widehat{P}$ (after finding all the candidate sensors) that has the maximum benefit with respect to M. Also, for each sensor m in M, the algorithm computes its optimal incremental benefit (as defined above), and picks the sensor $\widehat{m}$ that has the highest optimal incremental benefit. If the optimal incremental benefit of $\widehat{m}$ is higher than the benefit of selected $\widehat{P}$, then $\widehat{m}$'s sensing radius is increased to its optimal incremental radius, otherwise the candidate path $\widehat{P}$ is added to M. That completes one stage of the algorithm. The above process is repeated until the given query region is completely covered by M.

ALGORITHM 1. Centralized Greedy Algorithm
Input: A sensor network and a query region R_Q.
Output: A set of connected sensor cover M. Each with assigned sensing and transmission radius.

BEGIN
Let M denote the set of sensors selected.
Let I be a node whose minimum sensing region intersects R_Q.
$S(I) =$ Minimum sensing radius S_1;
$T(I) = 0$;
$M := I$;
while (R_Q is not covered by M)
    Let SP be the set of candidate paths, and $\widehat{P} \in SP$ be the candidate path with maximum benefit;
    Let $\widehat{m} \in M$ be the sensor node with most optimal incremental benefit;
    $BP =$ Benefit of $\widehat{P}$;
    $Bm =$ Optimal incremental benefit of $\widehat{m}$;
    if ($BP > Bm$)
        $M = M \cup \widehat{P}$
    else
        Set $S(\widehat{m})$ to $\widehat{m}$'s optimum incremental radius.
    end if;
end while;
RETURN M**;**
END ◇

The above described Algorithm 1 can be implemented in $O(n^3)$ time, where n is the size of the network. The following theorem proves the near-optimality of the solution delivered by the algorithm. We omit the proof, as a formal proof will be presented later when we generalize this algorithm to the variable radii connected k-cover problem.

Definition 20. (Link Radius) The *link radius* is defined as the maximum communication distance between any two sensors whose maximum sensing regions intersect. □

Theorem 8. *Algorithm 1 returns a connected sensor cover whose energy cost is at most $O(r(1 + \log d))|OPT|$, where r is the link radius of the sensor network, d is the maximum number of subelements in any sensing region, and $|OPT|$ is the energy cost of an optimum solution. Since, $d = O((nh)^2)$ ([Gupta et al. 2003]), the solution delivered by Algorithm 1 is within $O(r \log(nh))$ factor of the optimal solution. Recall that h is the total number of sensing radius choices available to a sensor node.* □

Distributed Greedy Algorithm (DGA). We now briefly describe the distributed version of the Algorithm 1 proposed in the previous section. The distributed algorithm presented here is similar to the distributed approximation algorithm proposed in [Gupta et al. 2003] for constructing a connected sensor cover. The Distributed Greedy Algorithm (DGA) works in stages, and at each stage, a *candidate path* is added to the already selected sensor set M, or the sensing range of a sensor in M is increased, until the whole query region is covered by M. Throughout the algorithm, the following variables are maintained:

—M, the set of sensors that have already been selected.
—SP, the set of candidate paths.
—$\widehat{P}$, the most recently added candidate path.
—$\hat{C}$, the candidate sensor associated with $\widehat{P}$.

Each stage of the the distributed algorithm consists of four phases as described below.

—Candidate Path Search (CPS). In this phase, the most recently added candidate sensor $\hat{C}$ broadcasts a CPS message within a range of $2r$ *communication distance*. In this broadcast phase, each sensor broadcast the CPS message with the maximum transmission range.

—Candidate Path Response(CPR). Any sensor that receives the CPS message checks whether it is a new candidate sensor (by checking whether its maximum sensing region intersects with any sensor in $\widehat{P}$). If so, it sends a CPR message (along with the associated candidate path formed by the routing path took by the CPS message) to $\hat{C}$, the originator of the CPS message.

—Selection of Best Candidate Path/Sensor. After gathering all CPR message, the sensor $\hat{C}$ calculates the benefit of each of the candidate paths and picks the candidate path $\widehat{P}_{new}$ (and the corresponding candidate sensor $\hat{C}_{new}$) that has the highest benefit. Moreover, it computes the optimal incremental benefit of each sensor in M, and picks the sensor $\widehat{m} \in M$ that has the maximum optimal incremental benefit. If the benefit of $\widehat{P}_{new}$ is greater than the optimal incremental benefit of $\widehat{m}$, then the sensor $\hat{C}$ unicasts all the required parameters to $\hat{C}_{new}$ after adding $\widehat{P}_{new}$ to M, and the $\widehat{P}_{new}$ and $\hat{C}_{new}$ now become the new (and current) $\widehat{P}$ and $\hat{C}$ respectively. If the optimal incremental benefit of $\widehat{m}$ is greater than the benefit of $\widehat{P}_{new}$, then the sensor $\hat{C}$ unicasts all the required parameters to

$\widehat{m}$, which becomes the new (and current) $\widehat{P}$ and $\hat{C}$. Also, $\widehat{m}$'s sensing radius is increased to attain the optimal incremental benefit.

—Repeat. The new $\hat{C}$ broadcasts the CPS messages again and initiates a new stage. This continues, until a leading sensor $\hat{C}$ decides that the sensing region R_M successfully covers the whole query region R_Q.

We make similar optimization as in [Gupta et al. 2003] to reduce the communication cost incurred by the distributed algorithm. In Section 7, we show that the solution returned by the above described Distributed Greedy Algorithm is very close to that returned by the Centralized Greedy Algorithm (Algorithm 1).

Greedy Algorithm for Connected Sensor k-Cover. We now extend the Centralized and Distributed Greedy Algorithms to the variable radii connected sensor k-cover problem. For generalization to k-coverage, we need to define a more general notion of benefit, namely k-benefit.

Definition 21. (k-Value of a Sensor Set) Given a sensor network and a query region, the k-value of a set of sensors M (with assigned radii) is denoted as $V(M,k)$ and is defined as the sum of the total number of times (bounded by k) each valid subelement is covered by the sensors in M. More formally, the k-value of a set M of sensors, $V(M,k)$, is computed as:

$$V(M,k) = \sum_{t \in T} min(k, \sum_{s \in M} \delta(t,s)),$$

where T is the set of valid subelements, and $\delta(t,s)$ is 1 if the subelement t is covered by the sensor s, and else 0. □

Definition 22. (k-Benefit of Candidate Path) Consider a candidate path P and set of already selected sensors M. The k-Benefit of P with respect to M is defined as $(V(M \cup P, k) - V(M,k))/(E(M \cup P) - E(M))$, where $E(\mathcal{I})$ is the total energy cost of a set of sensors $\mathcal{I}$. □

Definition 23. (Optimal Incremental k-Benefit) Let M be the set of sensors already selected by the greedy algorithm, and m be a sensor node in M with an assigned sensing radius of $S(m)$. The *i*ncremental k-Benefit of increasing m's sensing radius from $S(m)$ to $S'(m)$ is defined as the increase in k-value of the set M divided by the increase in energy cost of m. The sensing radius $S'(m)$ of m that results in the maximum incremental k-benefit is defined as the *o*ptimal incremental radius of m with respect to M, and the corresponding incremental benefit is called the *o*ptimal incremental k-benefit of m. □

Now, the Centralized Greedy Algorithm can be generalized for variable radii connected sensor k-cover as follows. At each stage of the algorithm, we either add a candidate path with maximum k-benefit, or increase the sensing radius of a sensor $m \in M$ that has the highest optimal incremental k-benefit. The Distributed Greedy Algorithm is similarly generalized. Below, we show that the generalized Centralized Greedy Algorithm still delivers a connected sensor k-cover that is within $O(r \log hn)$ factor of the optimal energy cost, where h is the total number of sensing radius choices available to a sensor node.

THEOREM 9. *The generalized Centralized Greedy Algorithm returns a connected sensor k-cover whose energy cost is at most $O(r(1+\log d))E(OPT)$, where r is the link radius of the sensor network, d is the maximum number of subelements in any sensing region, and $E(OPT)$ is the energy cost of an optimum solution OPT. Since, $d = O((nh)^2)$ ([Gupta et al. 2003]), the solution delivered by the greedy algorithm is within $O(r\log(hn))$ factor of the optimal solution.*

PROOF. We call a valid subelement as *active*, and say that it contains t *active* copies at a given stage of the greedy algorithm, if it is covered by $k-t$ $(t>0)$ distinct sensors in the greedy solution at that stage.

Let us consider a sensor I in the optimal solution OPT. Let $\theta(I)$ be the assigned sensing region of I and $\mathcal{F}(I)$ be the sensing energy cost of I in the optimal solution OPT. Let A_{Ij} denote the number of active copies of subelements within $\theta(I)$ after the j^{th} round of the greedy algorithm. Let M_j be the set of sensors selected by the greedy algorithm after the j^{th} round, and l be the total number of iterations of the greedy algorithm. Then, $E(M_j) - E(M_{j-1})$ is the energy cost added to the greedy solution during the j^{th} round. We uniformly distribute this added energy cost as a charge over all the active copies of subelements in the optimal solution. Thus, the charge accumulated on a sensor $I \in OPT$ with $\theta(I) \neq \phi$ (i.e., $A_{I0} > 0$) is:

$$C(I) = \sum_{j=1}^{l} (A_{I(j-1)} - A_{Ij}) E_j / V_j,$$

where $E_j = E(M_j) - E(M_{j-1})$ and $V_j (= V(M_j,k) - V(M_{j-1},k))$ is the total number of new active valid subelements copies covered by the greedy algorithm in the j^{th} round. Now, we know that $V_j/E_j \geq A_{I(j-1)}/k(r+\mathcal{F}(I))$ for $j \geq 2$, and $V_1/E_1 \geq (A_{I0} - A_{I1})/k(r+\mathcal{F}(I))$. This is because after some valid subelements inside $\theta(I)$ have been covered, sensor I with sensing region $\theta(I)$ becomes a candidate sensor, and a candidate path of energy cost at most $r+\mathcal{F}(I)$ and covering at least $A_{I(j-1)}/k$ active valid subelements is available for selection by the greedy algorithm. Thus, the total charge accumulated on a sensor $I \in OPT$ over the entire course of the greedy algorithm is at most:

$$C(I) \leq k(r+\mathcal{F}(I))(1 + \sum_{j=2}^{l} (A_{I(j-1)} - A_{Ij})/A_{I(j-1)})$$

Using some algebra, the above gives $C_I \leq k(r+\mathcal{F}(I))(1+\log A_{I0})$. Note the fact that by adding the charges accumulated by all such sensors in OPT, we actually charge each energy cost at least k times. Thus what we obtain is at least k times the energy cost of the solution returned by the greedy algorithm. Thus,

$$k * E(M_l) \leq k * ((r * |OPT_S| + \sum_{I \in OPT} \mathcal{F}(I)(1+\log d))$$

and

$$E(M_l) \leq (r * |OPT_S| + \sum_{I \in OPT} \mathcal{F}(I)(1+\log d)$$

where $|OPT_S|$ is the number of sensors in OPT with $\theta(I) > 0$. Note that $A_{I0} \leq kd$, where d is the maximum number of valid subelements in the maximum sensing region of any sensor node. Moreover, the total energy cost $E(OPT)$ of the optimal solution satisfies:

$$E(OPT) \geq \mathcal{G}_{min} * |OPT| + \sum_{I \in OPT} \mathcal{F}(I)$$

where G_{min} is the minimum transmission energy cost of a sensor, and $|OPT|$ is the number of sensors in the optimal solution OPT. Thus,

$$E(M_l)/E(OPT) \leq (r/\mathcal{G}_{min}) * (1 + \log d).$$

Here, $\mathcal{G}_{min}$ is a constant for a particular type of sensor. As stated in [Gupta et al. 2003], d is within $O(n^2h^2)$, recall h is the number of sensing radius choices in a sensor. Thus, the total energy cost $E(M_l)$ of the solution returned by the greedy algorithm is within $O(r \log hn)$ factor of the optimal energy cost. ∎

7. PERFORMANCE EVALUATION

We built a specific simulator for the distributed algorithms, and carried out extensive experiments to evaluate the performance of the proposed algorithms. The simulator randomly places sensors within a given region. The simulator does not model any link layer protocol or wireless channel characteristics. Thus, all messages in the simulator are transmitted in an error-free manner. While such a simulator models an idealized communication subsystem, it is sufficient for our purpose of comparing the performance of our proposed algorithms.

Energy Cost Model. The sensing energy cost function depends on the specific sensor type and environment, but is usually of the form $S(I)^x$, where $S(I)$ is the assigned sensing radius and x is a constant [Pattem et al. 2003]. Similarly, the transmission energy cost function is of the form $T(I)^y$, where $T(I)$ is the assigned transmission radius and y is a constant between 2 to 4 [Wan et al. 2001]. For our experiments, we chose $x = y = 4$. The energy consumption of idling radio and processor for each sensor is usually constant. Assuming each active sensor sends same amount of data during each time slot, the total energy cost incurred in keeping a sensor node active for a slot time is:

$$E(I) = \alpha S(I)^4 + (1-\alpha)T(I)^4 + C, \tag{1}$$

where α is a parameter that signifies relative weight of sensing and transmission energies. In our experiments, we use three different values of α viz. 0.1, 0.5, and 0.9 to simulate different sensor types. Essentially, when α is 0.1, the energy consumption due to sensing is relatively much less than the energy consumption due to transmission. We measure the performance of our algorithms for all these three energy cost models.

Network and Battery Parameter Values. We run our experiments with the following choice of parameter values. The maximum sensing radius S^* as well as the maximum transmission radius T^* for each sensor node is chosen to be 10. Each sensor can choose from 5 different sensing and transmission radius: 2, 4, 6, 8, or 10. We randomly distribute a certain number of sensor nodes in a query region of size 50×50. The total size n of sensor network is between 100 to 600, representing scarce to significantly dense sensor network density. In our experiments, we set each sensor node's battery power as 12,000,000 units, and the constant C in the energy cost function is set at 2,000 units. If the sensing and transmission radii of a sensor node are set to the maximum (10), the total energy cost incurred in keeping the node active for a unit time is 12,000 units. In a naive approach wherein all sensor nodes are kept active with maximum sensing and transmission radii, the sensor network will last for 1,000 time units, for any value of α.

Algorithms. We compare the performance of the following algorithms in our experiments for the variable radii 1-connected k-cover problem.

—Voronoi Based Algorithm – The localized distributed algorithm described in Section 5.2.

—Centralized Greedy Algorithm (CGA) – the greedy approximation algorithm described in Algorithm 1.

—Distributed Greedy Algorithm (DGA) – the distributed version of Algorithm 1 described in Section 6.

—Centralized Greedy Algorithm for Fixed Radii (CGA_FIXED) – the centralized greedy algorithm proposed in [Gupta et al. 2003] for the fixed radius connected sensor cover. Here, we try all 25 combinations of sensing and transmission radii (from 2, 4, 6, 8, and 10 units), and pick the best solution among them. The distributed version of the algorithm is denoted as DGA_FIXED. Note that DGA_FIXED can be extended to 1-connected k-cover problem in the way as described in [Zhou et al. 2004].

For the Voronoi Based algorithm, we use polygon clipping [Foley et al. 1990] to construct the voronoi diagrams. In addition, to save communication costs, we estimate sleeping benefit $B(I)$ of a node I using only the local voronoi diagram of I (i.e., we assume that I has the same local voronoi diagrams as its local voronoi neighbors).

Since none of the above algorithms except the Voronoi Based algorithm applies to the general k_1-connected k_2-cover problem, we compare the performance of Voronoi Based algorithm for the general k_1-connected k_2-cover problem with the three heuristics listed below in this case.

(1) COMPLETE_KCONE – This is a straightforward method. All sensors keep active. V-R assignment in Section 4 is used to assign sensing radii; Cone based topology control [Bahramgiri et al. 2002] is used to assign transmission radii.

(2) COMPLETE_KRNG – All sensors keep active. V-R assignment in Section 4 is used to assign sensing radii; k-RNG is used to assign transmission radii.

(3) SLEEPING_FIXED – In this method, the sleeping benefits are calculated and nodes are turned inactive exactly same as in the Voronoi Based algorithm. The only difference is that the radii are fixed. Here, we try all 25 combinations of various sensing and transmission radii (from 2, 4, 6, 8, and 10 units), and pick the best solution among them.

Note that the DGA_FIXED and SLEEPING_FIXED algorithms are only for comparison purposes – it is infeasible for the network nodes to collaboratively decide on the best combination of fixed radii.

Cost Model for Message Transmissions. During the construction phase (execution of an algorithm to construct a VRCSC), the energy cost incurred in transmitting a message is proportional to the size of the message. Specifically, we assume the energy cost incurred in transmitting a message of size ℓ bytes during the construction phase is

$$(1-\alpha)T^{*}\ell/100,$$

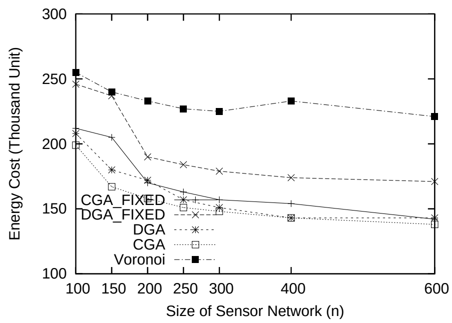
wherein T^{*} is the maximum transmission range. Thus, during the construction process, we use the maximum transmission range. Note that T^{*} is 10 for all algorithms except for the DGA_FIXED and SLEEPING_FIXED algorithm, wherein T^{*} is the fixed transmission range being used for *that* combination. The above equation indicates that even for the same construction process, more energy is consumed on sensor networks with smaller value of α.

Experiments. We have conducted six sets of experiments. The first set of experiments is to compare the performance of the various algorithms in terms of the total energy cost of the connected sensor cover delivered by the algorithm. Second and third sets extend the comparison to variable radii 1-connected k-cover problem. In particular, the second set shows the total energy cost of the connected 3-cover for varying network size n, and the third set presents the results for varying coverage degree k. In the fourth set of experiments, we compare the performance of the various distributed algorithms (DGA, DGA_FIXED, Voronoi) for variable radii 1-connected k-cover problem in terms of their effectiveness in prolonging the sensor network lifetime. We define the *network lifetime* as the number of data gatherings that can be achieved using a sequence of connected sensor covers. Each data gathering results in consumption of one battery unit from each sensor in the connected sensor cover used for the data gathering. The fifth set of experiments compares appropriate algorithms in terms of the total energy cost of the 3-connected 2-cover set delivered by them. Finally, in the last set of experiments, we compare appropriate algorithms in terms of their effectiveness in prolonging lifetime of the sensor network through a 3-connected 2-cover. Each data point in each of the graph plots shown is an average over five experiments.

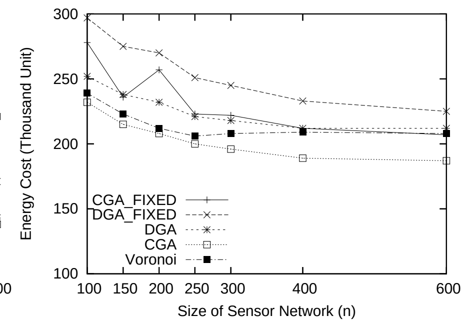
Energy Cost of Connected Sensor Covers. As shown in Figure 6, we can see that for the 1-connected 1-cover problem, the Centralized Greedy Algorithm (CGA) delivers the solution with least total energy cost among all algorithms, and DGA performs very close to CGA. In general, the Voronoi Based algorithm also performs quite close to the CGA and DGA algorithm, except for the case when $\alpha = 0.1$ (i.e., when transmission energy cost has a higher weightage) – implying that the RNG approach of assigning transmission radii can potentially be improved further.

For the 1-connected k-cover problem (see Figure 7 and 8), we see that the algorithms proposed for the variable radii consistently deliver better results than CGA_FIXED and DGA_FIXED, with the performance difference increasing with the increase in α or the coverage degree k. More importantly, the Voronoi Based algorithm continues to perform close to CGA and DGA algorithms in most cases, except for low α and k.

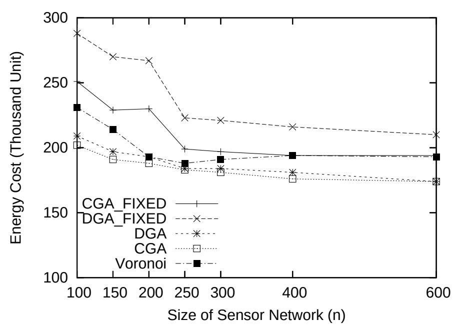
Network Lifetime using Connected Sensor Covers. Figure 9 and 10 show



(a) $\alpha = 0.1$



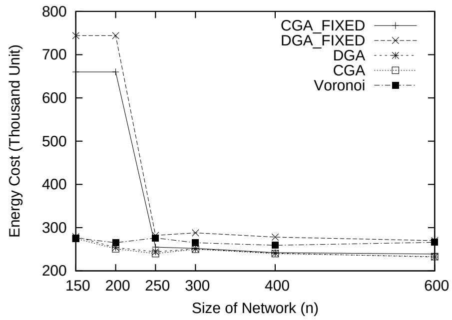
(b) $\alpha = 0.5$



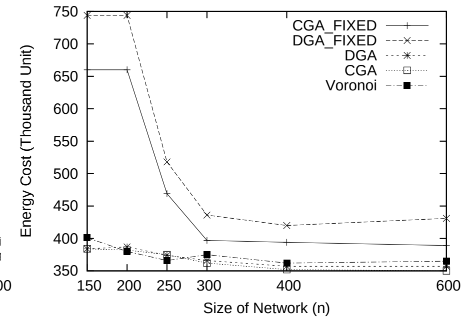
(c) $\alpha = 0.9$

Fig. 6. Total energy cost of the connected sensor cover delivered by various algorithms. Here, as defined in Equation 1, α is a parameter that signifies relative weight of sensing and transmission energies.

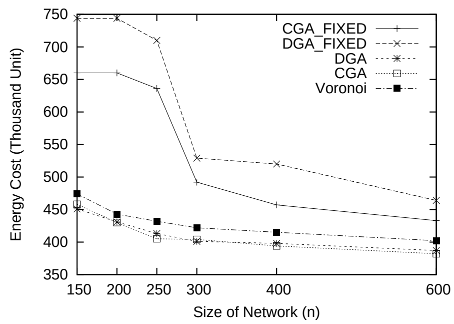
that our approaches also prolongs the lifetime of the sensor network. Due to the small size of messages in the Voronoi Based algorithm compared to DGA, the Voronoi Based algorithm has a much lower transmission energy overhead during the construction phase. This is particularly true when α is small and hence, the message transmission cost is relatively expensive. Hence, the Voronoi Based algorithm performs much better than the other distributed algorithms (DGA and DGA_FIXED) in terms of prolonging the network lifetime when α is 0.1 or 0.5. In the case when $\alpha = 0.9$, the performance of the algorithms is primarily dominated by the sizes of the solution returned. Thus, for $\alpha = 0.9$, the Voronoi Based algorithm and DGA perform close to each other, while outperforming the DGA_FIXED approach. For dense networks and low α, DGA performs worse than DGA_FIXED due to much higher construction cost. This is because at the end of each stage of DGA and DGA_FIXED, a fairly large message containing the entire state information (proportional to the size of the network) is transmitted, and the transmission



(a) $\alpha = 0.1$



(b) $\alpha = 0.5$



(c) $\alpha = 0.9$

Fig. 7. Total energy cost of 1-connected 3-cover delivered by various algorithms.

range used in DGA_FIXED for message transmissions may be less that that used by DGA. The above performance gap between DGA and DGA_FIXED is less pronounced in Figure 10, since DGA_FIXED is forced to use a high radii combination to construct a connected 3-cover. As mentioned before, it is impractical to implement DGA_FIXED and is shown only for comparison purposes.

Energy Cost of the 3-Connected 2-Cover. In Figure 11, we present the energy cost of the 3-connected 2-cover returned by the algorithms for varying network density. As the COMPLETE_KCONE and COMPLETE_KRNG heuristics keep all sensors active, their solutions incur more energy cost than Voronoi Based algorithm. This is particularly true, when the network density is high. Between these two non-sleeping schemes, COMPLETE_KRNG is consistently more energy efficient than COMPLETE_KCONE. Because they both keep all the sensors active while employing the same scheme in assigning sensing radii, this saving in energy cost for COMPLETE_KRNG over COMPLETE_KCONE is purely from transmission power control, resulting from Theorem 4. We can see that as the relative weight of transmission cost increases (α decreases), the difference between COM-

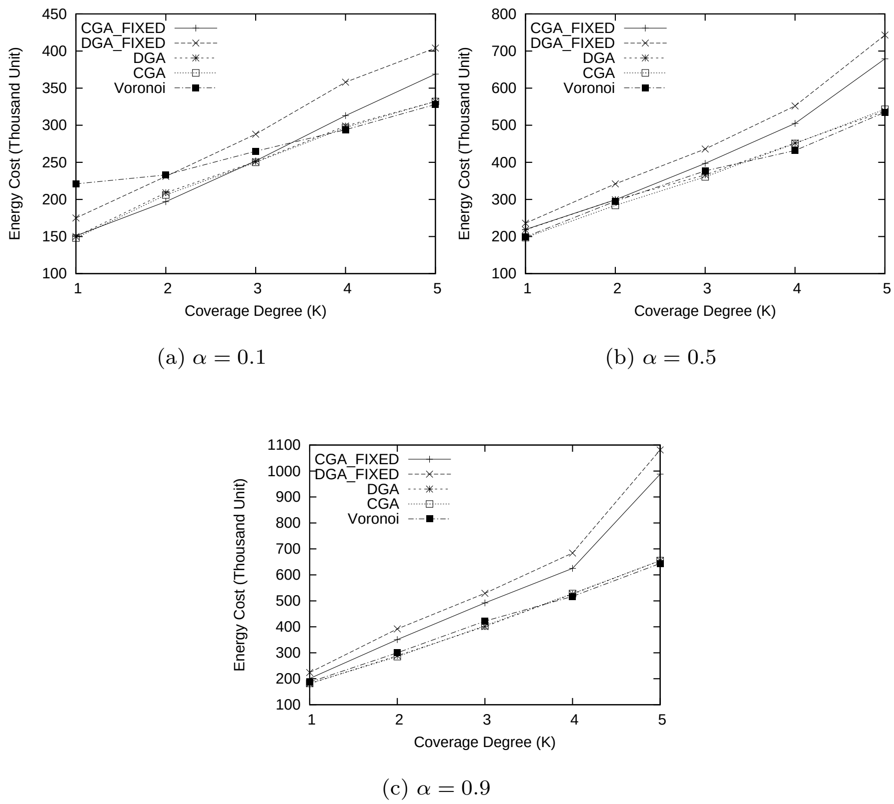
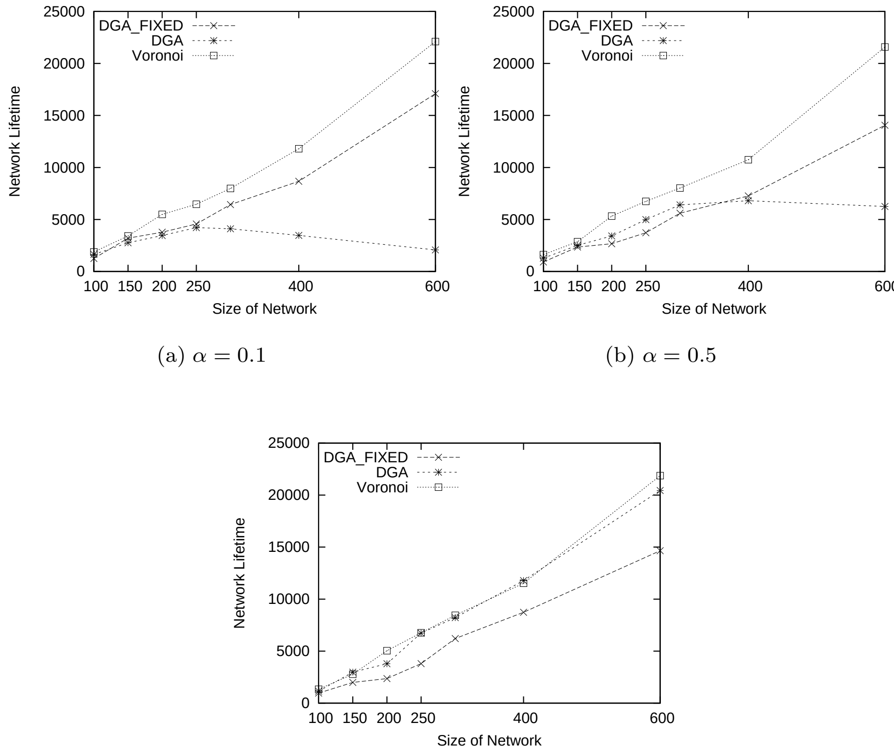


Fig. 8. Total energy cost of 1-connected k-cover delivered for varying k. Here, the network size (n) is 300.

PLETE_KRNG and COMPLETE_KCONE grows rapidly.

The SLEEPING_FIXED heuristic performs better than COMPLETE_KRNG when the network density is relatively high, in which case, a significant part of sensors can be put to sleeping and thus energy cost can be saved. While this saving is less obvious when the network density is low. When the network density is low, a much less percentage of sensors can satisfy the sleeping condition. As a result, both COMPLETE_KRNG and SLEEPING_FIXED have similar number of active sensors. In this situation, COMPLETE_KRNG shows superior performance over SLEEPING_FIXED because of the elaborate power control scheme it employed. This explains the crossover of the performance trends of the two schemes in the figures.

Note that the results shown here for SLEEPING_FIXED are the best one picked from all combinations of available transmission and sensing radii levels. Still, our Voronoi Based algorithm still consistently beats these best fixed radii results. This demonstrates the need for adaptive ability to control transmission and sensing



(a) $\alpha = 0.1$

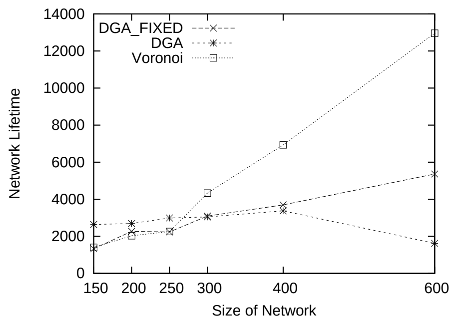
(b) $\alpha = 0.5$

(c) $\alpha = 0.9$

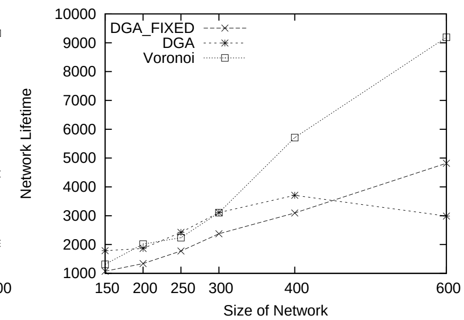
Fig. 9. Sensor network lifetime using a connected sensor cover delivered by various distributed algorithms.

ranges for energy conservation.

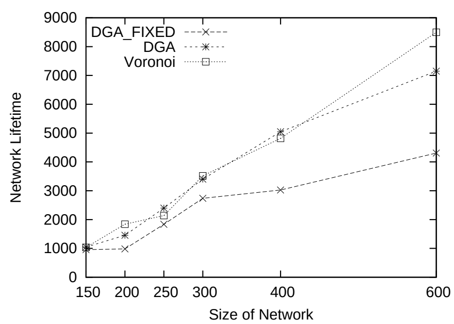
Network Lifetime using 3-Connected 2-Cover. We run these algorithms to generate a 3-connected 2-cover, which remain active until some sensor dies. Then in COMPLETE_KCONE and COMPLETE_KRNG, the neighboring nodes reassign their sensing and transmission ranges to compensate for this; while in Voronoi Based algorithm and SLEEPING_FIXED, the dying sensor awakens its neighboring sleeping sensors to sustain the 3-connected 2-cover. The awakening of the sleeping sensors can be done in a local manner, which is as described in section 4. As mentioned before, when employing the naive method, the network can last 1000 units of time under the settings. In Figure 12, we see that the energy efficiency exhibited in the connected sensor cover set really leads to a prolonged network lifetime. Again, Voronoi Based algorithm prolongs the network lifetime more effectively than all the others. COMPLETE_KCONE exhibits worst network lifetime than the others, which can be explained by the fact that its generated sensor cover incurs significantly more energy cost than the others. Again, COMPLETE_KRNG



(a) $\alpha = 0.1$



(b) $\alpha = 0.5$



(c) $\alpha = 0.9$

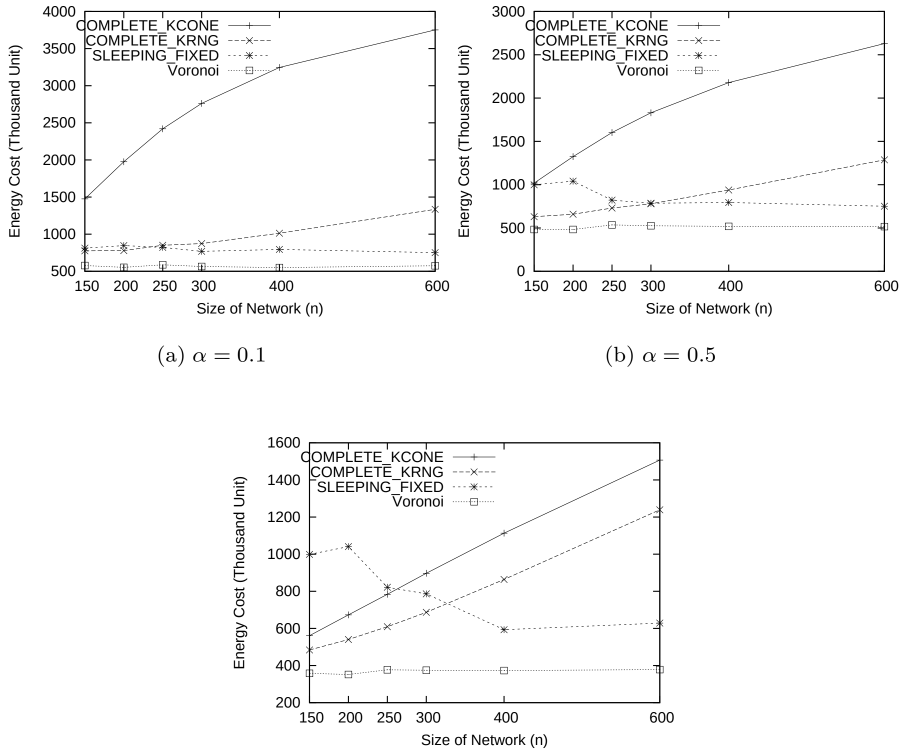
Fig. 10. Sensor network lifetime using a connected 3-cover delivered by various distributed algorithms.

and SLEEPING_FIXED show a similar trend as in the previous set of experiments on energy cost. When the network is sparse, COMPLETE_KRNG is better; while when the network is dense, SLEEPING_FIXED shows better performance. Also, we can see Voronoi Based algorithm performs well in exploiting the network redundancy. It greatly improves the network lifetime as the network size (redundancy) grows.

Summary of Simulation Results. The following observations summarize the results of our simulations comparing the performance of various algorithms.

—In general, the variable radii algorithms perform much better than the best possible fixed radii algorithms, and Voronoi based algorithm significantly outperforms other distributed algorithms in terms of network lifetime.

—For the 1-connected k-cover problem, we make the following important observations. (a) CGA returns the lowest energy-cost solution, with the Voronoi Based algorithm performing close to CGA for most parameter values, (b) The variable radii algorithms (CGA and DGA) return lower energy-cost solutions than their



(a) $\alpha = 0.1$

(b) $\alpha = 0.5$

(c) $\alpha = 0.9$

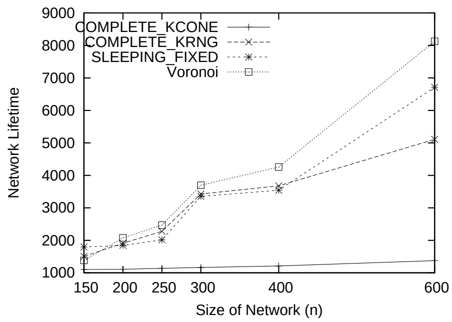
Fig. 11. Total energy cost of a 3-connected 2-cover for varying network size.

fixed radii counterparts (CGA_FIXED and DGA_FIXED), and (c) Voronoi Based algorithm outperforms both DGA and DGA_FIXED in terms of network lifetime.

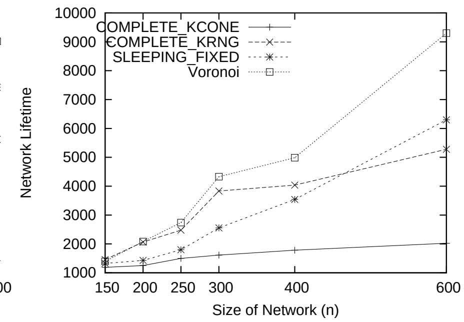
—For the k_1-connected k_2-cover problem, CGA is inapplicable. Thus, we compare our Voronoi Based algorithm with three simple heuristics viz., COMPLETE_KCONE, COMPLETE_RNG, and SLEEPING_FIXED. We observe that the Voronoi Based Algorithm significantly outperforms all the other heuristics in terms of energy cost of the delivered solution as well as the network lifetime.

8. CONCLUSIONS

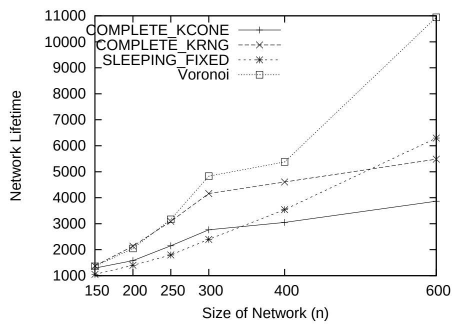
Given that sensor networks are typically redundant, we have presented an approach to conserve energy by exploiting redundancy in the network. In particular, we addressed the problem of constructing a connected sensor cover in a sensor network model wherein each sensor can control/adjust its sensing and transmission power/range. For the above problem we proposed various centralized approximation and communication-efficient distributed algorithms. We extended these algorithms to a more general connected sensor k-cover problem. Moreover, the Voronoi Based algorithm was extended to the most general k_1-connected k_2-coverage problem. Through extensive experiments, we demonstrated the usefulness of our approaches in prolonging the network lifetime. In particular, our proposed localized



(a) $\alpha = 0.1$



(b) $\alpha = 0.5$



(c) $\alpha = 0.9$

Fig. 12. Sensor network lifetime using a 3-connected 2-cover delivered by various distributed algorithms.

Voronoi Based algorithm is shown to perform very well in comparison with other approaches.

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